University of Göttingen
Cycle in SP4 "Stochastic processes"
Prof. Dr. Anja Sturm
Start: Winter semester 2021/2022

Focus: Stochastic processes are collections of random variables. In this lecture course, the index set of this collection of random variables will generally be a subset of the real numbers (discrete or continuous) representing time. Thus, a stochastic process in discrete or continuous time describes the random evolution of the state of a system over time.

In this cycle of lectures we consider the theory of stochastic processes as well as their applications. Therefore, after studying various classes of stochastic processes and characterisation methods we will get to know stochastic models that are based on stochastic processes and that are used in the Natural Sciences, in particular in Physics and Biology, as well as in the Social Sciences, such as in Economics and Finance.

Prerequisites: Lecture course on measure and probability theory ("Maß- und Wahrscheinlichkeitstheorie") or equivalent. Prior attendance of the lecture "Stochastik" would be ideal but is not strictly a prerequisite.

Lecture times and location: The introductory lecture in the winter semester 2021/2022 will be Monday and Thursday from 10-12. The lecture will be online.

Exercises: There will be a weekly exercise session for which solutions to an exercise sheet will have to be handed in.

Seminar: In the winter semester 2021/2022 an accompanying seminar on stochastic processes will be offered.

Details on the lecture syllabus:

Cycle 1: winter semester 2021/22
Markov processes, martingales, Brownian motion
Basics of stochastic processes, existence, uniqueness, filtrations, Markov processes, in particular Markov chains in discrete and continuous time, random walks, recurrence and transience, invariant distributions and convergence, description of general continuous time Markov processes via generators and (Feller) semigroups, Poisson processes and (Poisson) point processes, discrete and continuous time martingales, stopping times, martingale inequalities, martingale convergence theorems, optional stopping theorem, Brownian motion, construction, strong Markov property, sample path properties, 0-1-laws, reflection principle, law of iterated logarithm, outlook towards stochastic (Itô) integration with respect to Brownian motion and martingales.

Cycle 2: winter semester 2022/23
Continuous time martingales, stochastic Itô integration, stochastic differential equations
Finer properties of Brownian motion and martingales in continuous time, regularity theorems, Doob-Meyer decomposition, square integrable martingales and quadratic variation, Lévy processes, stochastic Itô integration, Itô formula, martingale representation theorem, Girsanov theorem, stochastic differential equations, strong and weak existence and uniqueness of solutions, diffusions, application to in particular Biology and Finance.

Cycle 3: summer semester 2023
Convergence of stochastic processes, general Markov process theory
Weak convergence and convergence in distribution in general metric spaces, Prohorov's theorem, convergence of right continuous paths on the Skorohod space $D[0,\infty)$, in particular Donsker's theorem, convergence of random walk to Brownian motion, characterisation of general continuous time Markov processes via generators and (Feller) semigroups, martingale problems, existence and uniqueness, diffusion approximations, applications in particular in the areas of population processes, branching processes, interacting particle systems as well as queueing theory.

Cycle 4: winter semester 2023/24
Stochastic models
In this last part of the cycle we consider a variety of stochastic models and applications, in particular in the area of particle systems that play a role in im Biology, especially in Genetics and Ecology, in the study of infectious disease, and in Physics and Chemistry. In order to describe the evolution of populations, amongst others, we will study classical branching processes alongside their ancestral structure which are given by coalescent processes. These models are also an important tool for analysing random graphs. We will further consider spatial interacting particle systems such as the so called contact process, the voter model and percolation.

References

[1] A. Klenke. *Wahrscheinlichkeitstheorie.* Springer, 2011.

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[3] J.R. Norris *Markov Chains.* Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press, 1998.

[4] P. Bremaud. *Markov Chains: Gibbs Fields, Monte Carlo Simulation, and Queues.* Springer, 2010.

[5] I. Karatzas and S. E. Shreve. *Brownian Motion and Stochastic Calculus*, volume 113 of *Graduate Texts in Mathematics.* Springer, 1991.

[6] D. Revuz and M. Yor. *Continuous martingales and Brownian Motion.* Springer, 1991.

[7] S. N. Ethier and T. G. Kurtz. *Markov Processes: Characterization and Convergence.* Wiley Series in Probability and Mathematical Statistics. Wiley, 1986.

[8] T. M. Liggett *Continuous time Markov processes.* American Mathematical Society, Graduate Studies in Mathematics, Volume 113, 2010.

[9] K. B. Athreya and P. E. Ney *Branching processes.* Dover books, 1972.

[10] T. G. Kurtz. *Approximation of Population Processes.* CBMS-NSF Regional Conference Series in Applied Mathematics. SIAM, 1981.

[11] T. M. Liggett *Stochastic Interacting Systems: Contact, Voter and Exclusion Processes.* Springer Grundlehren der Mathematischen Wissenschaften 1999.

[12] R. van der Hofstad *Random graphs and complex networks.* Lecture notes, Eindhoven University of Technology, 2011.